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ENERGY TRANSFER AND TRIADIC INTERACTIONS IN COMPRESSIBLE TURBULENCE*

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Abstract. Using a two-point closure theory, the Eddy-Damped-Quasi-Normal-Markovian (EDQNM) approximation, we have investigated the energy transfer process and triadic interactions of compressible turbulence. In order to analyze the compressible mode directly, the Helmholtz decomposition is used. The following issues were addressed: (1) What is the mechanism of energy exchange between the solenoidal and compressible modes, and (2) Is there an energy cascade in the compressible energy transfer process? It is concluded that the compressible energy is transferred locally from the solenoidal part to the compressible part. It is also found that there is an energy cascade of the compressible mode for high turbulent Mach number ($M_t \geq 0.5$). Since we assume that the compressibility is weak, the magnitude of the compressible (radiative or cascade) transfer is much smaller than that of solenoidal cascade. These results are further confirmed by studying the triadic energy transfer function, the most fundamental building block of the energy transfer.

Key words. Compressible turbulence, energy transfer, triadic interactions, closure theories.

Subject classification. Fluid Mechanics

1. Introduction. It is well known that compressible turbulence plays a prominent role in a wide range of important scientific and engineering applications, including high speed transport, supersonic combustions, and acoustics. Recently, a large body of publications have been devoted to study various aspects of compressible turbulence using both direct numerical simulations (DNS) and large eddy simulations (LES). Here we simply mention a few representative works, such as Feiereisen et al. (1981), Passot and Pouquet (1985), Lee et al. (1991), Sarkar et al. (1991), Erlebacher et al. (1992), Kida and Orszag (1990), Blaisdell et al. (1993) and Porter et al. (1994). For a comprehensive review, the reader is referred to Lele (1995). These numerical simulations have substantially improved our understanding of compressible turbulence. Nevertheless, some basic physical processes of compressible turbulence, such as the energy transfer and triadic interactions, have not been explored even at low turbulent Mach number. For example, do we expect an energy cascade process of compressible velocity modes? How does the energy exchange process between the solenoidal and compressible modes take place? These type of studies require a substantial spectral scale range of interactions. As a result, it is very hard to utilize the DNS databases since these simulations are limited to very low Reynolds numbers and have only very limited spectral ranges. While LES can provide

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databases at substantially higher Reynolds numbers, subgrid models must be introduced.

Another way to generate high Reynolds number databases is by using two-point closure models. Direct Interaction Approximation (DIA) of Kraichnan (1959) is a well-established approach. Many authors have studied incompressible turbulence using DIA (see Leslie (1973)). The method of Eddy-Damped-Quasi-Normal-Markovian (EDQNM) models (Orszag (1970)) has been shown as a simpler, but effective, alternative to DIA. Recently, Bertoglio et al. (1996) have presented DIA and EDQNM equations for a weakly compressible turbulence.

The aim of this paper is to use the EDQNM closure theory to study the energy transfer and triadic interactions of compressible turbulence. The paper is organized as follows: First, we review closure assumptions of EDQNM and present the resulting transport equations. Second, we perform a detailed analysis of the non-linear transfer terms. Finally, we investigate the most fundamental aspect of the energy transfer process, the incompressible and compressible triadic interactions.

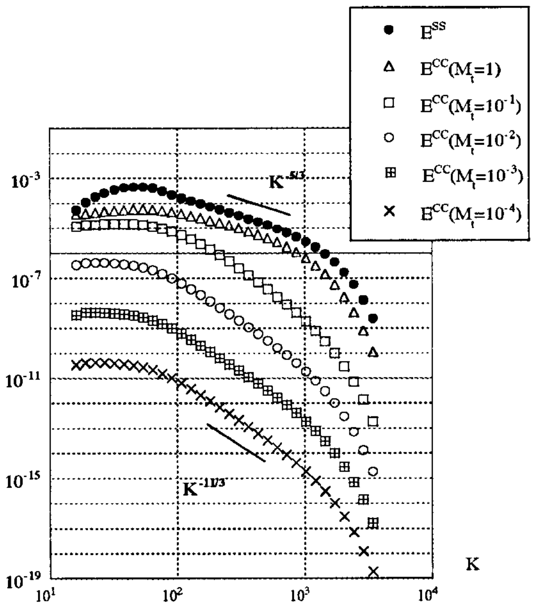


Figure 1: Solenoidal and compressible velocity correlations for different turbulent Mach numbers

FIG. 1.

2. Compressible EDQNM Model. The basic set of equations are the Navier-Stokes and continuity equations. The fluid is assumed to be homogeneous, isotropic and barotropic. Reynolds average and Fourier transform are used to obtain the fluctuating turbulent field equations in spectral space. The equations are partially linearized with respect to the density fluctuation which leads to the condition $M_t < 1$, where M_t is the turbulent Mach number defined as $M_t = \sqrt{q^2}/c_0$. q^2 is twice the turbulent kinetic energy, and c_0 is the sound speed. To analyze the compressibility effects, we use the Helmholtz decomposition to split the velocity vector into a solenoidal part $u^S(\mathbf{K}, t)$, which corresponds to the velocity fluctuations perpendicular to the wave vector $\mathbf{K}$ in the Fourier space, and a compressible part $u^C(\mathbf{K}, t)$, which corresponds to fluctuations in the direction of the wave vector.

The classical DIA approach (two point – two times) is used to derive compressible DIA equations. These equations are the starting point used to derive the compressible EDQNM equations (in two points and one time). The methodology is the same as that of the incompressible case (Leslie (1973) and Lesieur (1987)) but more equations are now needed. For more details, see Bertoglio et al. (1996) or Bataille (1994).

2.1. Spectral Equations. The EDQNM governing equations of weakly compressible turbulence are the following:

- an equation for the spectrum (E^{SS}), the auto-correlation of the solenoidal part of the velocity field:

$$\frac{\partial}{\partial t} E^{SS}(K,t) = -2\nu K^2 E^{SS}(K,t) + T^{SS}(K,t); \tag{1}$$

- an equation for the spectrum (E^{CC}), the auto-correlation of the "purely compressible" part of the velocity field:

$$\frac{\partial}{\partial t}\, E^{CC}(K,t) = -2\nu' K^2 E^{CC}(K,t) + T^{CC}(K,t) - 2c_0 K E^{CP}(K,t); \tag{2}$$

- an equation for the spectrum of the potential energy (E^{PP}) associated with the pressure:

$$\frac{\partial}{\partial t}\, E^{PP}(K,t) = 2c_0 K E^{CP}(K,t); \tag{3}$$

- an equation for the spectrum of the pressure-velocity correlation (E^{CP}):

$$\frac{\partial}{\partial t}\, E^{CP}(K,t) = -\nu' K^2 E^{CP}(K,t) + T^{CP}(K,t) + c_0 K(E^{CC}(K,t) - E^{PP}(K,t)). \tag{4}$$

In the case of a Stokesian fluid:

$$\nu' = \frac{\lambda + 2\mu}{<\rho>} = \frac{4}{3}\,\nu \tag{5}$$

μ and λ, two dynamic viscosities, are assumed to be uniform.

2.2. Energy Transfer Terms. In equations (1)-(4), T^{SS}, T^{CC}, and T^{CP} are the transfer terms. They contain several contributions:

$$T^{SS} = T_1^{SS} + T_2^{SS} + T_3^{SS} + T_4^{SS} + T_5^{SS} \tag{6}$$

$$T^{CC} = T_1^{CC} + T_2^{CC} + T_3^{CC} + T_4^{CC} + T_5^{CC} + T_6^{CC} \tag{7}$$

$$T^{CP} = T_1^{CP} + T_2^{CP} + T_3^{CP} + T_4^{CP} + T_5^{CP} + T_6^{CP} \tag{8}$$

Different contributions appearing in the transfer term acting on the solenoidal field are:

$$T_1^{SS} = \int_\Delta \frac{K^3}{PQ}\,\frac{1 - xyz - 2y^2z^2}{2}\,\theta_{KPQ}^{SS-SS-SS} E^{SS}(P,t)E^{SS}(Q,t)dPdQ \tag{9}$$

$$T_2^{SS} = \int_\Delta \frac{K^3}{PQ}\,\frac{(1-y^2)(x^2+y^2)}{1-x^2}\,\theta_{KPQ}^{SS-SS-CC} E^{SS}(P,t)E^{CC}(Q,t)dPdQ \tag{10}$$

$$T_3^{SS} = -\int_\Delta \frac{P^2}{Q}\,(xy+z^3)\theta_{KPQ}^{SS-SS-SS} E^{SS}(K,t)E^{SS}(Q,t)dPdQ \tag{11}$$

$$T_4^{SS} = \int_\Delta \frac{P^2}{Q} \, (2xy)\theta_{KPQ}^{SS-SS-CC} E^{SS}(K,t) E^{CC}(Q,t) dPdQ \tag{12}$$

$$T_5^{SS} = -\int_\Delta \frac{P^2}{Q} \, (z(1-z^2))\theta_{KPQ}^{SS-CC-SS} E^{SS}(K,t) E^{SS}(Q,t) dPdQ \tag{13}$$

Different contributions to the transfer term in the E^{CC} equation are:

$$T_1^{CC}(K,t) = \int_\Delta \frac{K^3}{PQ} \, ((x+yz)^2)\theta_{KPQ}^{CC-SS-SS} E^{SS}(P,t) E^{SS}(Q,t) dPdQ \tag{14}$$

$$T_2^{CC}(K,t) = \int_\Delta \frac{K^3}{PQ} \, \frac{(x^2-y^2)^2}{(1-x^2)} \, \theta_{KPQ}^{CC-SS-CC} E^{SS}(P,t) E^{CC}(Q,t) dPdQ \tag{15}$$

$$T_3^{CC}(K,t) = \int_\Delta \frac{K^3}{PQ} \, (x^2)\theta_{KPQ}^{CC-CC-CC} E^{CC}(P,t) E^{CC}(Q,t) dPdQ \tag{16}$$

$$T_4^{CC}(K,t) = -\int_\Delta \frac{P^2}{Q} \, 2z(1-z^2)\theta_{KPQ}^{CC-SS-SS} E^{CC}(K,t) E^{SS}(Q,t) dPdQ \tag{17}$$

$$T_5^{CC}(K,t) = -\int_\Delta \frac{P^2}{Q} \, (2z^3 - z + xy)\theta_{KPQ}^{CC-CC-SS} E^{SS}(Q,t) E^{CC}(K,t) dPdQ \tag{18}$$

$$T_6^{CC}(K,t) = \int_\Delta \frac{P^2}{Q} \, (2xy)\theta_{KPQ}^{CC-CC-CC} E^{CC}(K,t) E^{CC}(Q,t) dPdQ \tag{19}$$

And lastly, the contributions to the transfer term in the pressure velocity correlation are:

$$T_1^{CP}(K,t) = \int_\Delta \frac{K^3}{PQ} \, \frac{(x+yz)^2}{2} \, \theta_{KPQ}^{PC-SS-SS} E^{SS}(P,t) E^{SS}(Q,t) dPdQ \tag{20}$$

$$T_2^{CP}(K,t) = \int_\Delta \frac{K^3}{PQ} \, \frac{(x^2-y^2)^2}{2(1-x^2)} \, \theta_{KPQ}^{PC-SS-CC} E^{SS}(P,t) E^{CC}(Q,t) dPdQ \tag{21}$$

$$T_3^{CP}(K,t) = \int_\Delta \frac{K^3}{PQ} \, \frac{x^2}{2} \, \theta_{KPQ}^{PC-CC-CC} E^{CC}(P,t) E^{CC}(Q,t) dPdQ \tag{22}$$

$$T_4^{CP}(K,t) = -\int_\Delta \frac{P^2}{Q} \, z(1-z^2)\theta_{KPQ}^{PC-SS-SS} E^{CC}(K,t) E^{SS}(Q,t) dPdQ \tag{23}$$

$$T_5^{CP}(K,t) = -\int_\Delta \frac{P^2}{Q} \, \frac{2z^3 - z + xy}{2} \, \theta_{KPQ}^{PC-CC-SS} E^{SS}(Q,t) E^{CC}(K,t) dPdQ \tag{24}$$

$$T_6^{CP}(K,t) = \int_\Delta \frac{P^2}{Q} \, (xy)\theta_{KPQ}^{PC-CC-CC} E^{CC}(K,t) E^{CC}(Q,t) dPdQ \tag{25}$$

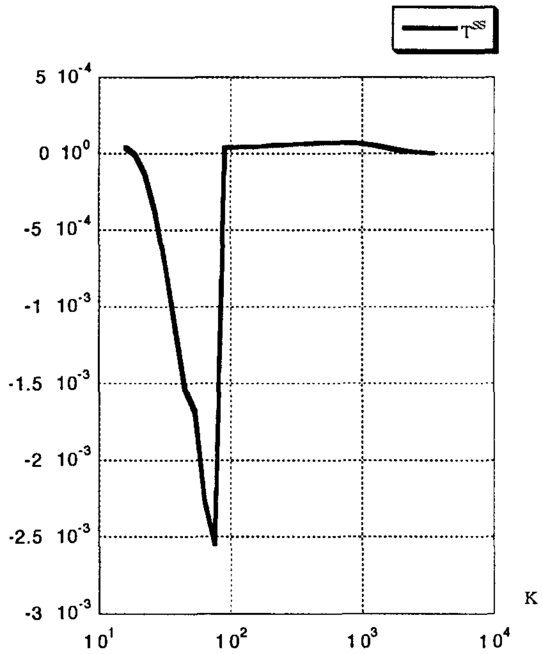


Figure 2:Solenoidal transfer term

FIG. 2.

The integration in the P, Q plane extends over a domain such that **K**, **P** and **Q** form a triangle. The expressions x, y, z are standard coefficients associated with the geometry of the triad and they the cosines of the angles respectively opposite to **K**, **P**, **Q**. Furthermore, temporal functions (defined by θ's) are the decorrelation times deduced by integrating the DIA equations over time. Their expressions can be found in Bertoglio et al. (1996).

The transfer terms appearing at the transport equations for the solenoidal and compressible modes will be analyzed in detail in the next section. Since there is no transfer term in the equation for the pressure-pressure correlation, this equation characterizes the exchange between the solenoidal mode and the compressible mode ($E^{CC} + E^{PP}$). Finally, the effect of T^{CP} is to interchange energy between E^{CC} and E^{PP} (Bataille and Bertoglio (1993)).

2.3. Energy Spectra. A force is applied to the large scales of the solenoidal velocity. Our analysis is carried out when both solenoidal and compressible modes have reached their asymptotic stationary states.

In Figure 1, we present the spectra of both solenoidal and compressible components of the velocity correlation corresponding after the asymptotic state is reached. The Taylor micro-scale Reynolds number, R_e, is approximately 140. Here $R_e \equiv \frac{q^2}{3}\sqrt{\frac{15}{\nu\epsilon}}$ and ϵ is the dissipation. In the figures, the dimensional unit of the correlation spectra is given as $m^3 s^{-2}$ and the dimensional unit of wavenumber K is defined as m^{-1}. The energy associated with the purely compressible mode is found to vary as the square of the turbulent Mach number. At low and moderate M_t, the spectrum of the compressible component shows a $K^{-11/3}$ behavior in the inertial range (Bataille and Bertoglio (1993) and Bertoglio et al. (1996)). This behavior has been confirmed with Large Eddy Simulation by Bataille et al. (1996).

In order to examine the dependence of the turbulent Mach number, it was allowed to vary from 10^{-4} to 1. Note, however, strictly speaking, our model is valid only for small M_t. Nevertheless, computations were carried out up to $M_t = 1$ in order to study the limiting behavior of the model.

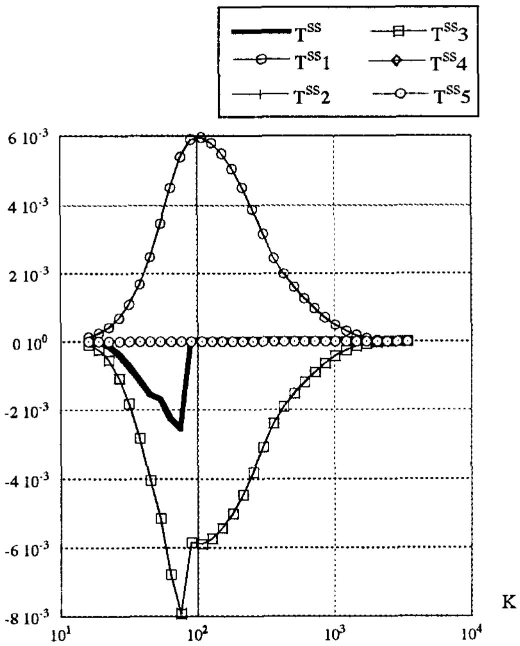


Figure 3:Contributions of the solenoidal transfer term

FIG. 3.

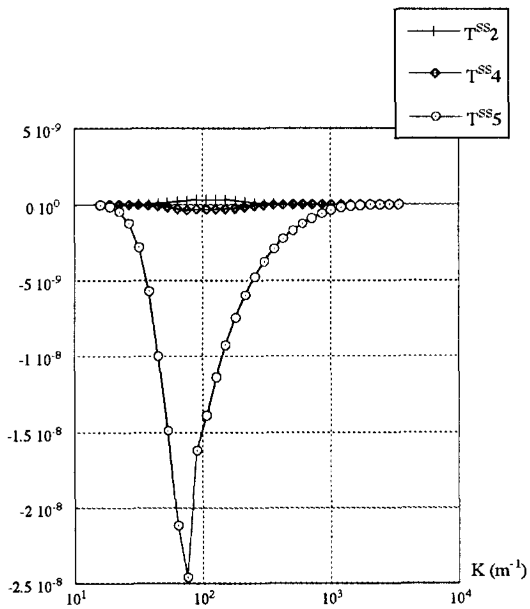


Figure 4:Contributions of the solenoidal transfer term

FIG. 4.

3. Study of the Transfer Terms.

3.1. Solenoidal Transfer Term. In Figure 2, we observe that T^{SS} has the usual shape observed in incompressible turbulence studies. Specifically, T^{SS} is negative in the large scales and positive for small scales. Physically this corresponds to the energy transfer from the large scales to the smaller ones.

There are 17 contributors to the transfer function of compressible turbulence. This should be be com-

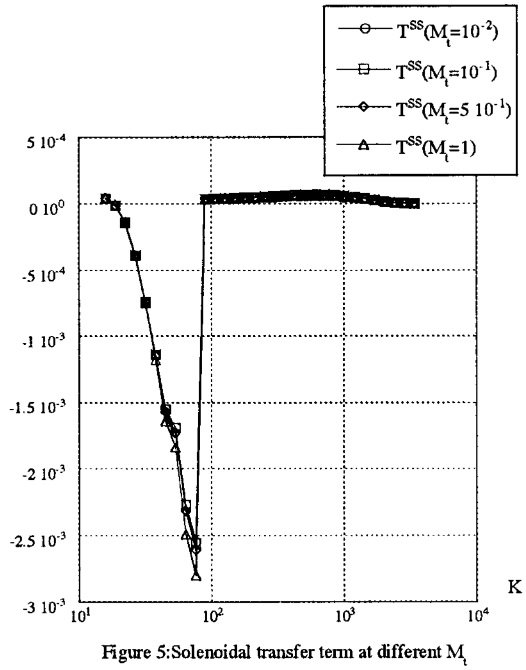


FIG. 5.

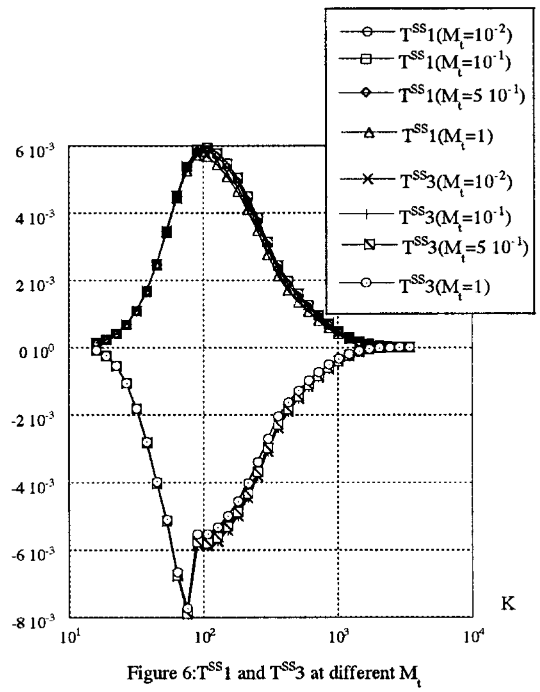


FIG. 6.

pared with its counterpart of incompressible turbulence:

$$T_s^{SS}(K,t) = T_1^{SS}(K,t) + T_3^{SS}(K,t). \tag{26}$$

Indeed, the two key contributors to T^{SS} are T_1^{SS} and T_3^{SS}, which are the same terms that one finds in incompressible turbulence. These terms are usually called "input" and "output" terms. For our weakly

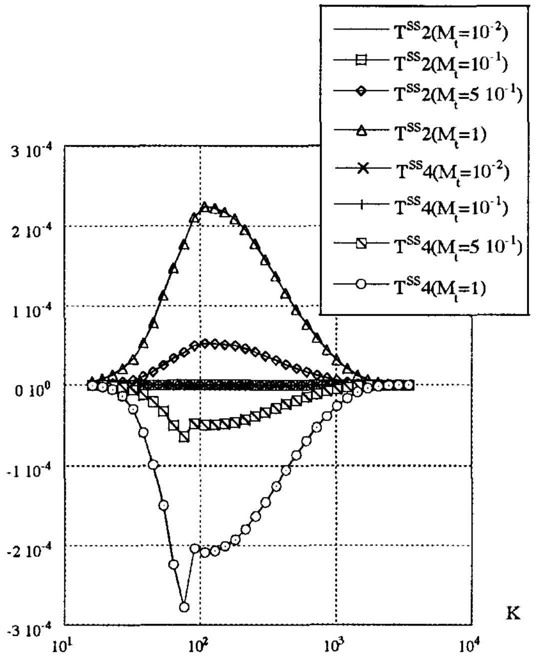


Figure 7: $T^{SS}2$ et $T^{SS}4$ at different M_t

FIG. 7.

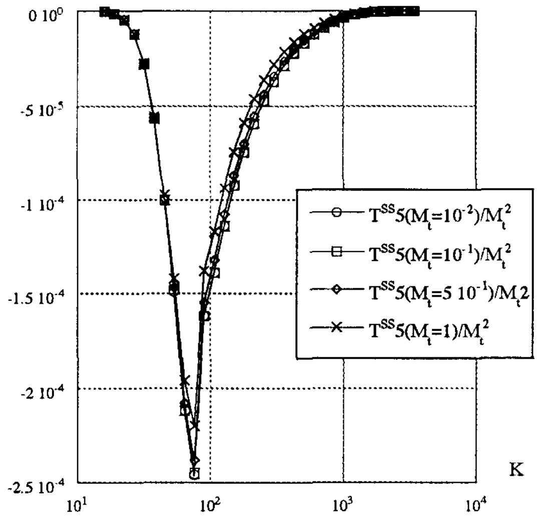


Figure8: $T^{SS}5$ divided by M_t^2

FIG. 8.

compressible turbulence, they are much more important contributors than the others (see Figure 3). Indeed, other contributors are negligible in comparison with T_1^{SS} and T_3^{SS}. We stress that the summation of these two terms is the net energy transfer of incompressible turbulence (solid line). To clearly illustrate the behaviors of the new terms, we plotted these "compressible" contributors (T_2^{SS}, T_4^{SS} and T_5^{SS}) in an enlarged scale (Figure 4) in order to observe their relative magnitudes. We found that T_5^{SS} is much larger than T_2^{SS} and T_4^{SS}, and consequently it makes the largest contribution to the compressible effects in T^{SS}. An important feature of this term is that it is negative for all spectral space, indicating that the energy transfer has been

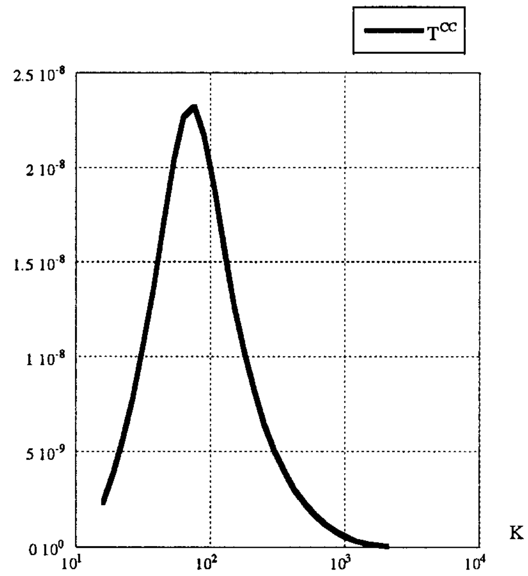


Figure 9:Compressible transfer term

FIG. 9.

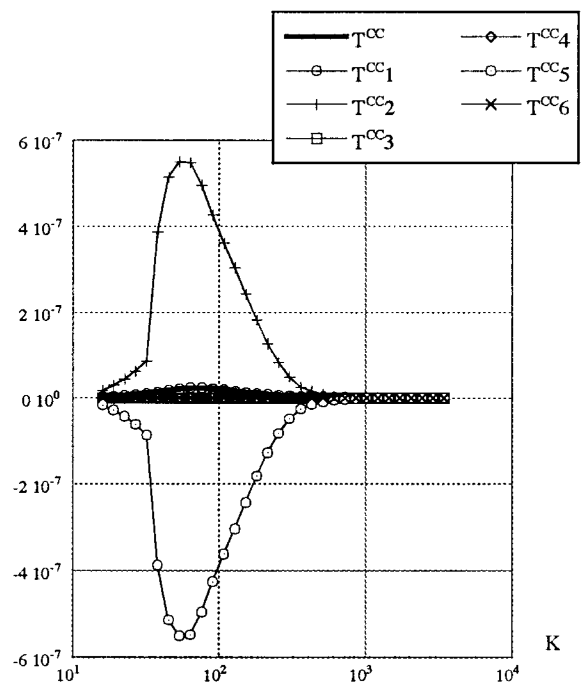


Figure 10:Contributions of the compressible transfert term

FIG. 10.

transferred from the solenoidal mode to the compressible mode.

We observe in Figure 5 that the compressibility has negligible effect on the solenoidal transfer term T^{SS}. The reason is that the dominant terms in T^{SS}, e.g., the incompressible contributions (T_1^{SS} and T_3^{SS}), are independent of M_t (Figure 6). On the other hand, the "compressible" contributions (which are smaller) depend strongly on the values of the turbulent Mach number. Figure 7 illustrates the dependences of T_2^{SS} and

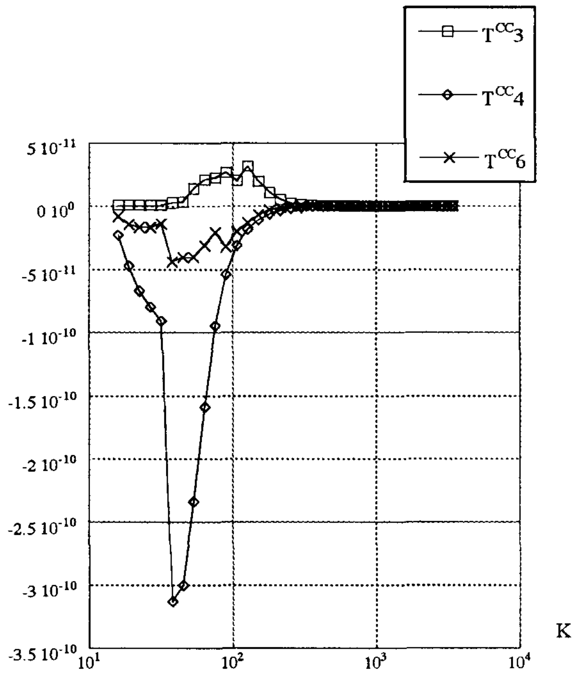


Figure 11:Contributions of the compressible transfert term

FIG. 11.

T_4^{SS} on the turbulent Mach number. We found that all the spectra of T_5^{SS} can be collapsed by dividing M_t^2 (Figure 8). Therefore, we conclude that T_5^{SS} is proportional to M_t^2, a result that can be found analytically. Since the dominant contributions in T^{SS} are insensitive to the variations of turbulent Mach number, T^{SS} is not affected by the compressibility.

3.2. Compressible Transfer Term. In this subsection, we will study the transfer term T^{CC}, and its individual contributors. This compressible transfer term appears in the transport equation of the compressible auto-correlation. As shown in Figure 9, the compressible transfer is positive for all spectral space. Hence, it is a term that is responsible for the production of compressible energy.

The different contributors to T^{CC} are plotted in Figure 10. It is clear that the two terms, T_2^{CC} and T_5^{CC}, are much larger than the others. Another term, T_1^{CC}, is a distant third in size. To illustrate the relative size of the smaller terms, we replotted these contributors in Figure 11 at the enlarged scale. Because two dominant terms have similar magnitudes but opposite signs, a strong cancelation between them is expected. Indeed for all cases considered, the term T_2^{CC} is always positive, whereas the other term T_5^{CC} is always negative. In fact, the cancelations are so complete, the summation of these two terms is now actually negligible in comparison with T_1^{CC} (Figure 12). The physical explanation for this 'almost perfect' cancelation is that T_2^{CC} and T_5^{CC} are the terms that take into account the interactions between slowly varying incompressible modes and two compressible modes (namely the interactions between acoustic waves and a solenoidal field). This interaction results in the production of acoustic energy on the same wave-number but now in another propagation direction. For an isotropic redistribution of acoustic energy, this effect of reorientation does not affect the spectral distribution of energy and leads to a zero net balance. Consequently, the most important term in the transport equation of E^{CC} is T_1^{CC}. Figure 13 demonstrates that this term is also much larger than all other contributors (T_3^{CC}, T_4^{CC} and T_6^{CC}).

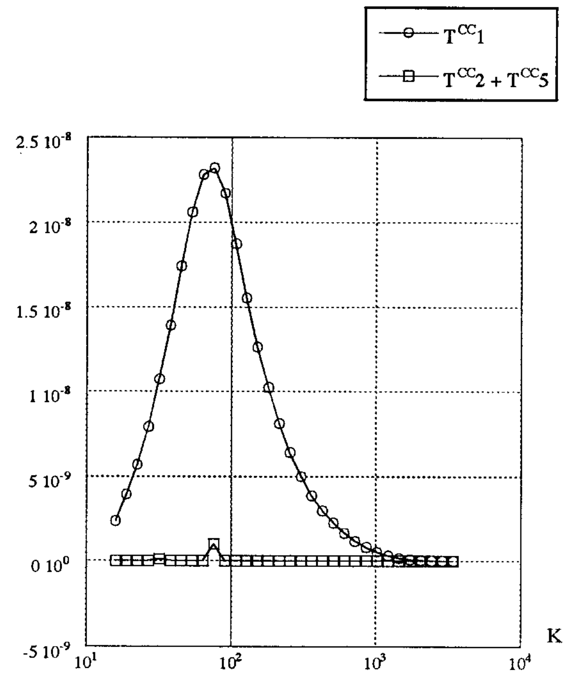


Figure 12:$T^{CC}1$ and ($T^{CC}2$+$T^{CC}5$)

FIG. 12.

Comparing $T_5^{SS}(K)$ to $T_1^{CC}(K)$, it is clear that they have similar magnitude but with an opposite sign (Figure 14). These two terms are essentially responsible of the energy exchanges between the solenoidal and compressible parts. Specifically, T_1^{CC} is the "input" energy term on the compressible mode whereas T_5^{SS} is the "output" term in the equation of the solenoidal mode. Based on these results obtained in spectral space, we conclude that there is a local transfer of energy from the solenoidal mode to the compressible mode. This result will be further confirmed by our analysis in the second part of the paper.

The total compressible transfer term T^{CC} is dependent on the compressibility (see Figure 15). As expected, its magnitude increases with the increasing of turbulent Mach number. We note that there is a shift of the peak spectrum towards the large K. Based on the properties of $T_5^{SS}(K)$, we expect that its compressible counterpart $T_1^{CC}(K)$ should also scale as M_t^2. Since the spectra of T_1^{CC} divided by M_t^2 collapses (Figure 16), this term can not be responsible for the peak shift of T^{CC}. Figure 17 showed that the 'almost perfect' cancelation between terms $T_2^{CC}(K)$ and $T_5^{CC}(K)$ for all turbulent Mach numbers is considered. Figure 18 illustrates how the term $T_4^{CC}(K)$ depends on M_t. Figure 19 shows the dependence of $T_3^{CC}(K)$ and $T_6^{CC}(K)$ on the values of turbulent Mach number. Although these two terms maintain opposite signs for all spectral space, the magnitudes and shape of $T_3^{CC}(K)$ and $T_6^{CC}(K)$ are clearly different. The 'imperfect' cancelation between these two terms leads to a cascade type of compressible energy transfer (Figure 20). These two terms involve $(E^{CC})^2$ and are important contributors at high turbulent Mach numbers. The interaction among the compressible mode begins to have influence. This cascade mechanism will be investigated in the next section.

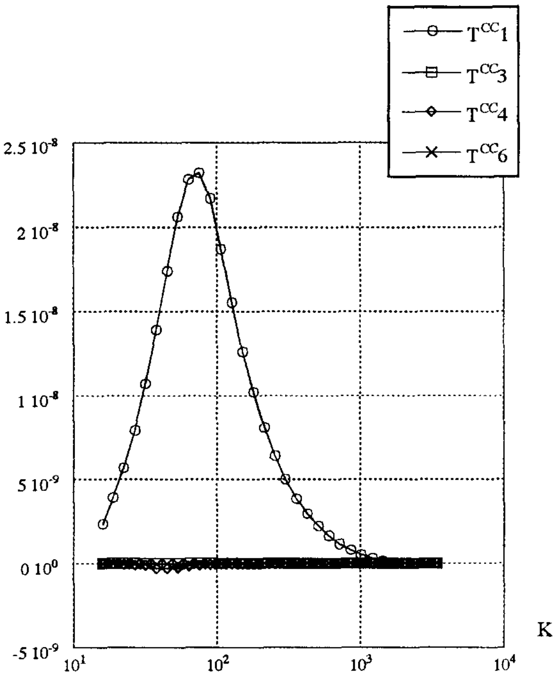


Figure 13:Contributions of the compressible transfert term

FIG. 13.

4. Triadic Interactions. The most fundamental building block of the energy transfer process is the triadic interactions. Specifically, we are interested in the energy transfer for a given mode **K** due to its interactions with all the pairs of modes **P** and **Q** = **K** - **P** that form a triangle with **K**. For this reason, we introduce the triadic energy transfer function, $T(K, P, Q)$, according to

$$T^{SS}(K) = \sum_{P,Q=|\mathbf{K}-\mathbf{P}|} T^{SS}(K, P, Q), and \tag{27}$$

$$T^{CC}(K) = \sum_{P,Q=|\mathbf{K}-\mathbf{P}|} T^{CC}(K, P, Q). \tag{28}$$

Here $T(K, P, Q)$ is defined as energy transfer to K due to triads with one leg in Q and the other in P. The average procedure is performed over spherical shell since the turbulence is isotropic.

An examination of the purely incompressible contributors ($T_S^{SS}(K, P, Q)$) reproduces the results of incompressible turbulence (Domaradzki and Rogallo (1990), Yeung et al. (1991, 1995), Ohkitani and Kida (1992), Zhou (1993a-b) and Zhou et al. (1996)) and again indicates that the purely solenoidal triadic energy transfer is not affected by compressible effects. The triadic solenoidal transfer $T^{SS}(K, P, Q)$ (with the compressible terms included) is essentially the same for a wide range of turbulent Mach number values (Figure 21). Although Figure 21 is only for $P = 512$ and $Q = 128$, we have examined other values of P and Q and found that our conclusion does not change with turbulent Mach number. As a result, the compressibility has very little influence on the solenoidal triadic interactions. Figure 22 is a typical plot showing how the structure of $T^{SS}(K, P, Q)$ changes with various Q values ($P = 512$, $M_t = 10^{-2}$). Again, this result is the same as that of incompressible turbulence.

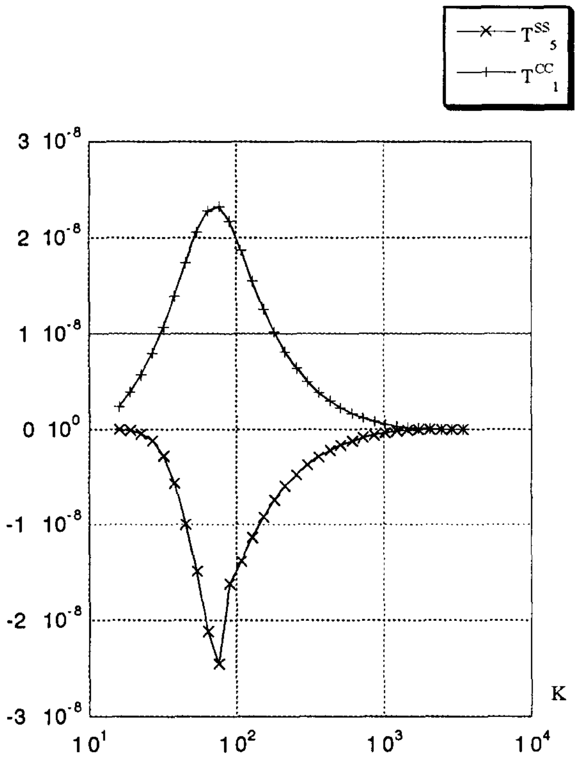


Figure 14:$T^{SS}5$ and $T^{CC}1$

FIG. 14.

We have found from the previous section that particular attention should be paid to the T_5^{SS} term since it is the term that is responsible for 'output' energy from solenoidal to compressible mode. As the Mach number increases, the magnitude of the 'output' energy increases but the basic structure remains (Figure 23). Since this term represents an energy output at a localized spectral region, we refer to it as the radiative (emission) triadic energy transfer. Figure 24 illustrates this type of interaction from various Q values at the given value of P ($P = 512$) for $M_t = 10^{-2}$. It is clear that the triadic interaction of this term is quite different from those of purely incompressible terms.

We now turn our attention to the triadic interactions in compressible energy transfer, $T^{CC}(K,P,Q)$. In Figure 25, we present $T^{CC}(K,P,Q)$ for various Q values when P is in the inertial range ($P = 512$). For this low Mach number 10^{-2}, we observe that the structures of $T^{CC}(K,P,Q)$ are rather similar for differing Q values. All of them show the *radiative* (absorption) type of energy transfer. Figure 26 shows that $T_1^{CC}(K,P,Q)$ is the dominant contributor to the compressible triadic energy transfer function. In fact, comparing Figures 24 and 26, we found that the absorption types of triadic energy transfer functions have the same magnitude but opposite sign as those of the emission type (T_5^{SS} term). Figure 27 further demonstrates that the triadic interactions $T_1^{CC}(K,P,Q) \approx -T_5^{SS}(K,P,Q)$ for all turbulent Mach numbers are under consideration. As a result, we conclude that all compressible energy has been transferred locally (in spectral space) from the solenoidal component.

From the previous section, we have found that the sum of $T_3^{CC}(K,P,Q) + T_6^{CC}(K,P,Q)$ is an important contributor to the compressible energy transfer. At low Mach number ($M_t = 10^{-2}$), terms $T_3^{CC}(K,P,Q) + T_6^{CC}(K,P,Q)$ are small and their triadic energy transfer terms show only a very weak energy cascade (Figure

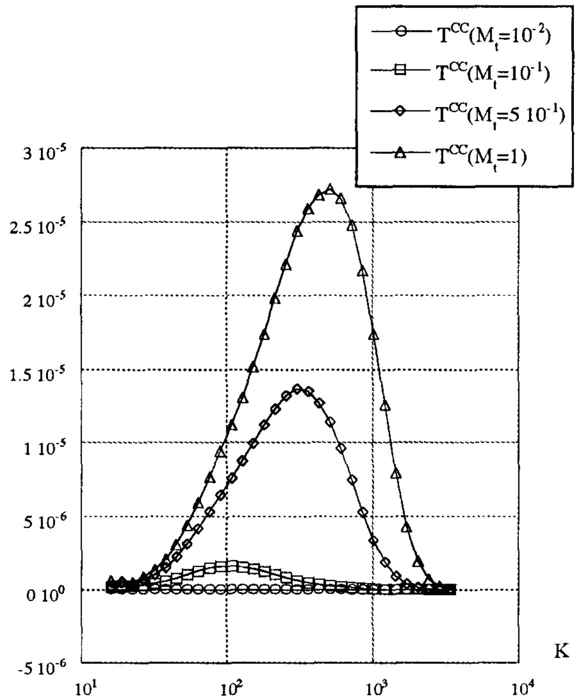


Figure 15:Compressible transfer term at different M_t

FIG. 15.

28). However, this situation changes rapidly as the Mach number increases. Indeed, the compressible energy cascade can be seen in Figures 29 and 30 where $T_3^{CC}(K,P,Q)+T_6^{CC}(K,P,Q)$ are plotted for several higher Mach numbers. From this analysis, we conclude that at high Mach number the cascade of compressible turbulence is a direct result of the fact $T_3^{CC}(K,P,Q)+T_6^{CC}(K,P,Q) > T_1^{CC}(K,P,Q)$. To further demonstrate this point, we plot the total compressible energy transfer term, $T^{CC}(K,P,Q)$, at different Mach numbers. It is clear that $T^{CC}(K,P,Q)$ changes its characteristic features from radiative to cascade as the turbulent Mach number increases (Figures 31 and 32). This is a result that can not be observed from the total compressible energy transfer function $T^{CC}(K)$.

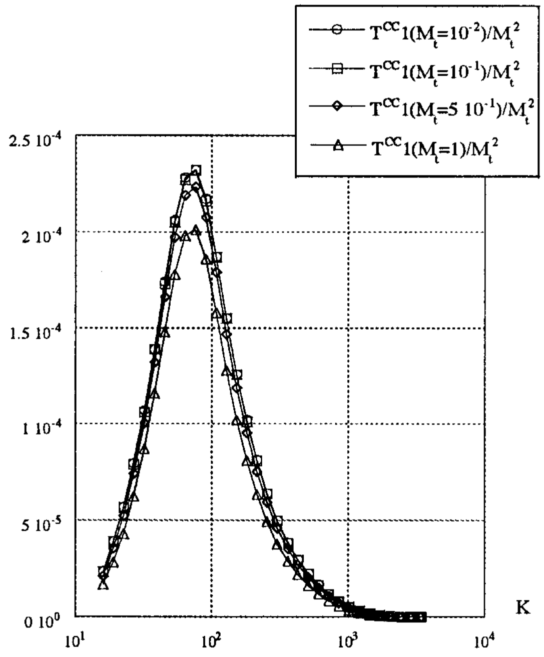


Figure 16:$T^{cc}1$ divided by M_t^2

FIG. 16.

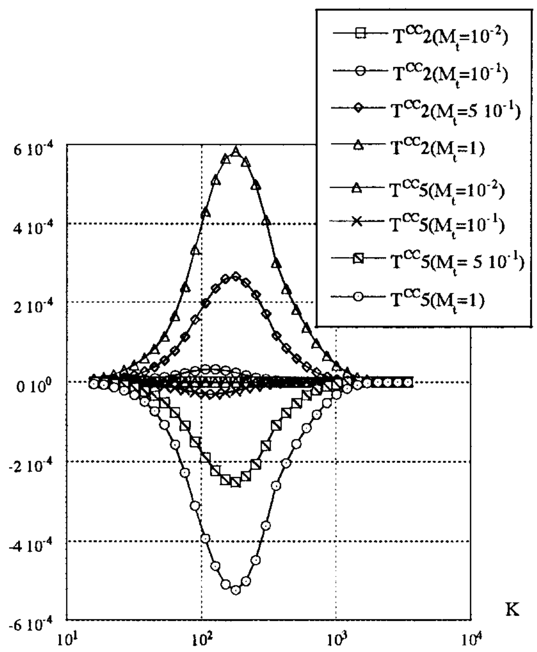


Figure 17:$T^{cc}2$ and $T^{cc}5$ at different M_t

FIG. 17.

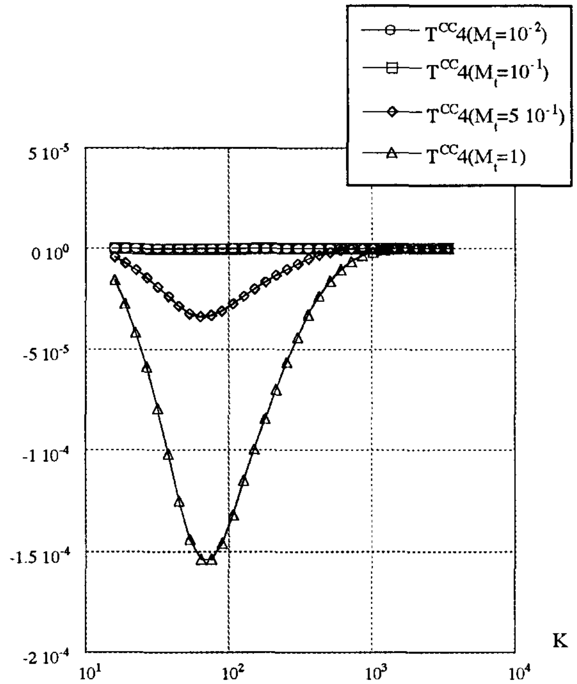


Figure 18:$T^{CC}4$ at different M_t

FIG. 18.

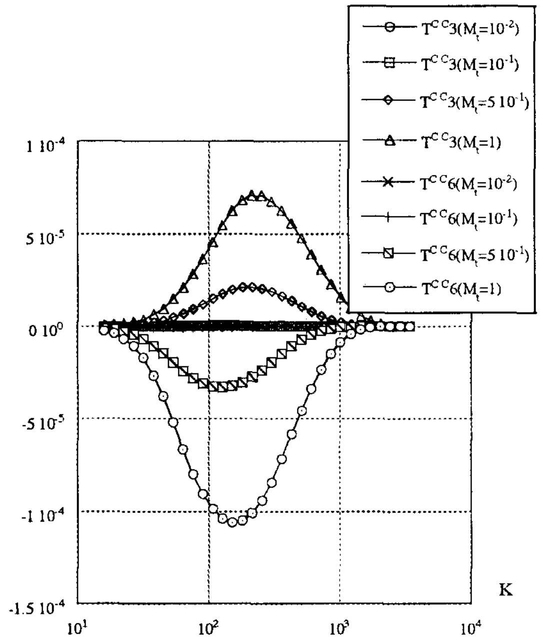
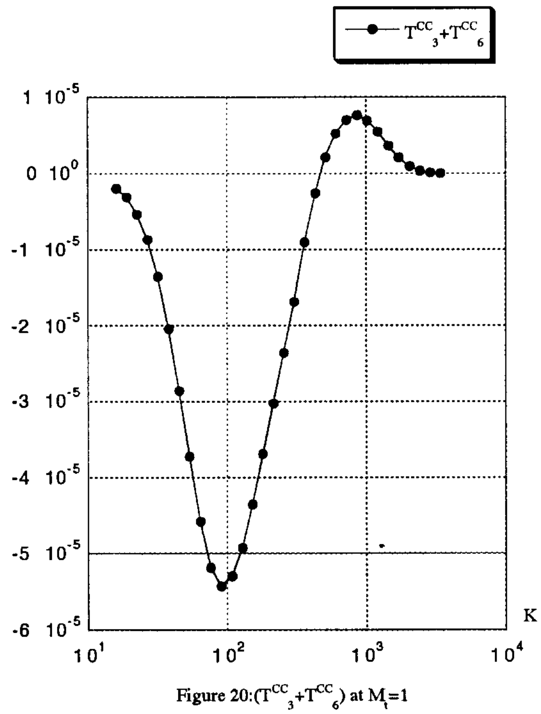


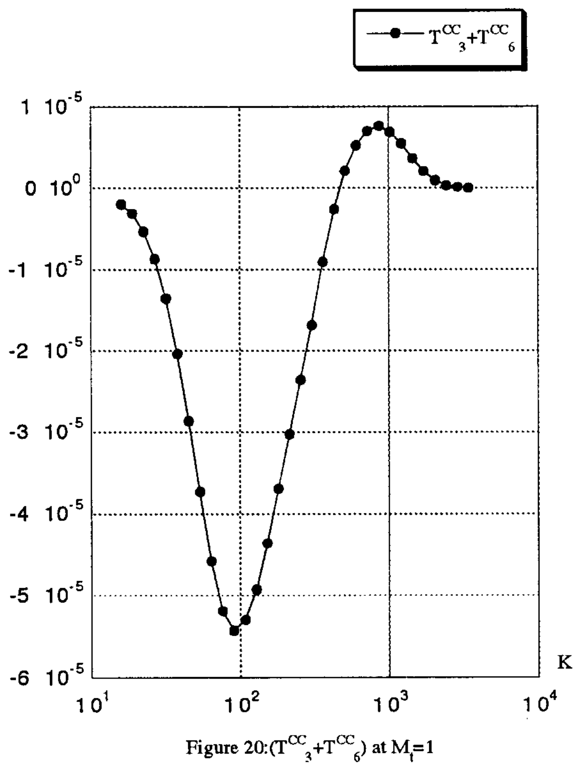
Figure 19:$T^{CC}3$ and $T^{CC}6$ at different M_t

FIG. 19.



$T^{CC}_3+T^{CC}_6$
1 10^-5
0 10^0
-1 10^-5
-2 10^-5
-3 10^-5
-4 10^-5
-5 10^-5
-6 10^-5
10^1
10^2
10^3
10^4
K
Figure 20:($T^{CC}_3+T^{CC}_6$) at M_t=1

FIG. 20.



$T^{CC}_3+T^{CC}_6$
1 10^-5
0 10^0
-1 10^-5
-2 10^-5
-3 10^-5
-4 10^-5
-5 10^-5
-6 10^-5
10^1
10^2
10^3
10^4
K
Figure 20:($T^{CC}_3+T^{CC}_6$) at M_t=1

FIG. 21.

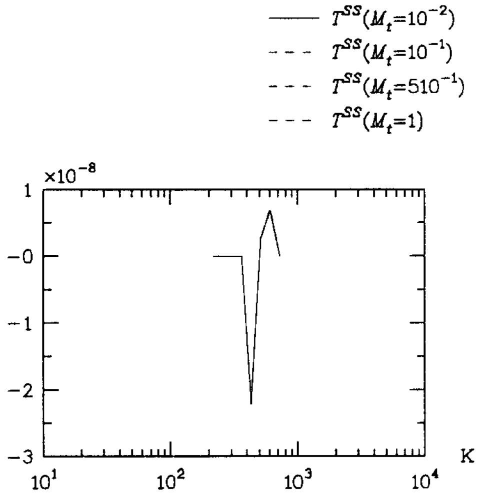


Figure 21:Triadic solenoidal transfer term at different M_t

FIG. 22.

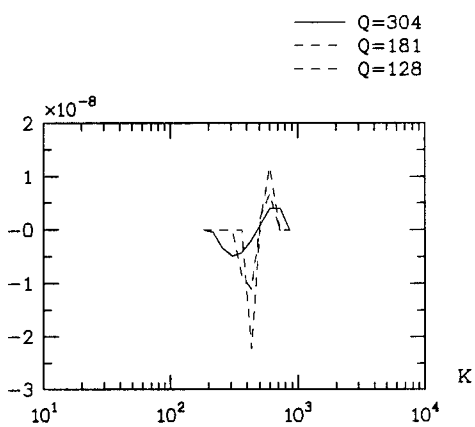


Figure 22:Triadic solenoidal transfer term for different Q

FIG. 23.

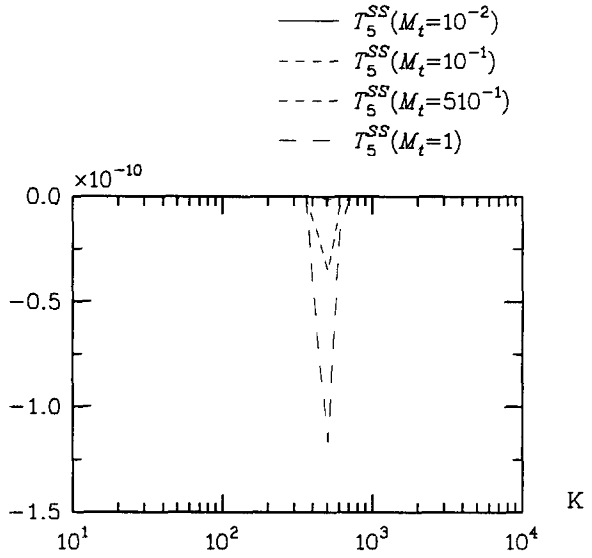


Figure 23:Triadic T_5^{SS} at different M_t

FIG. 24.

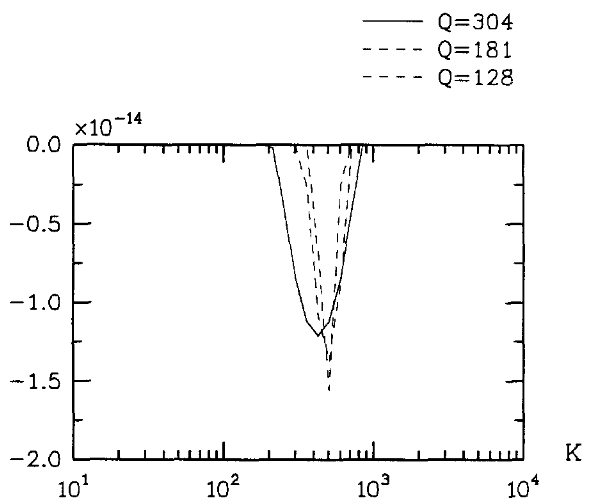


Figure 24:Triadic T_5^{SS} for different Q

FIG. 25.

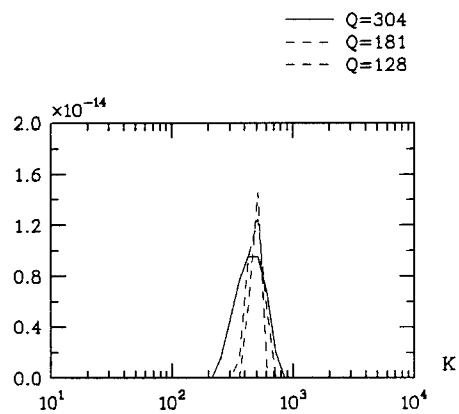


Figure 25:Triadic compressible transfer term for different Q

FIG. 26.

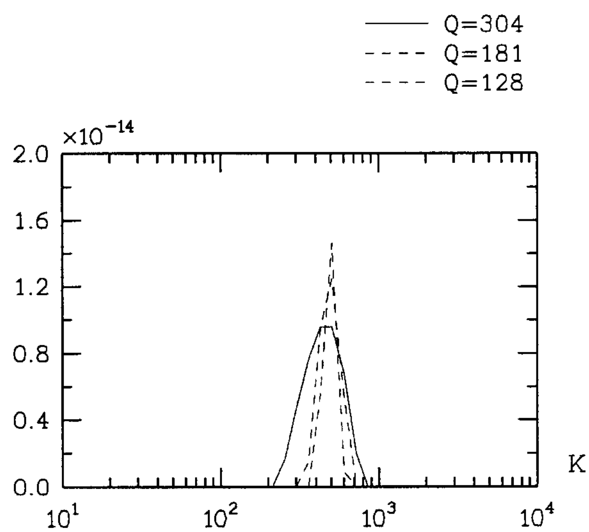


Figure 26:Triadic T_1^{CC} for different Q

FIG. 27.

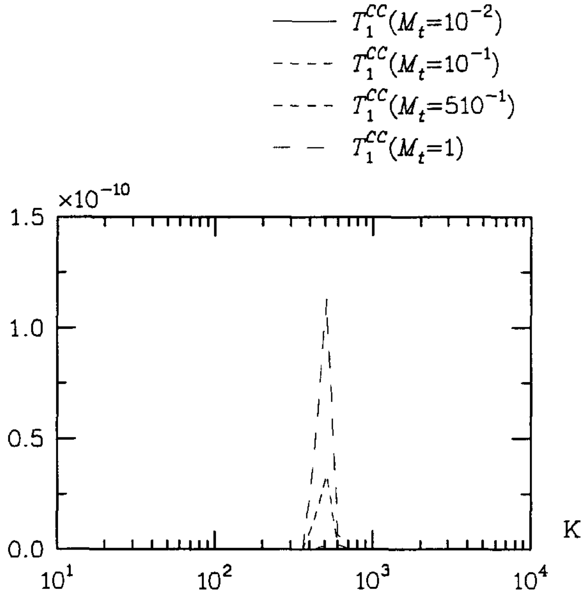


Figure 27:Triadic T_1^{CC} at different M_t

FIG. 28.

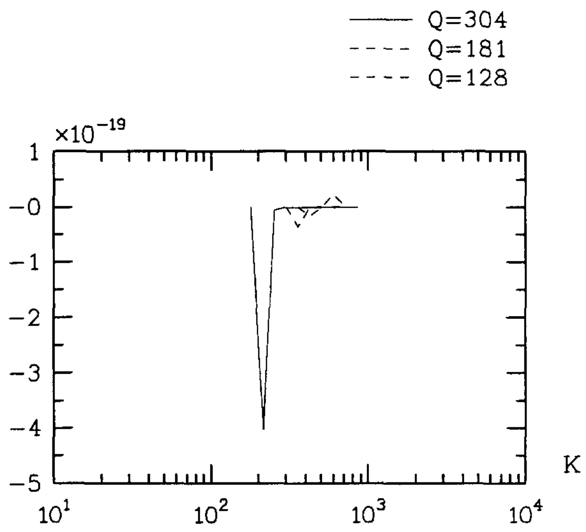


Figure 28:Triadic $T_3^{CC}+T_6^{CC}$ for different Q

FIG. 29.

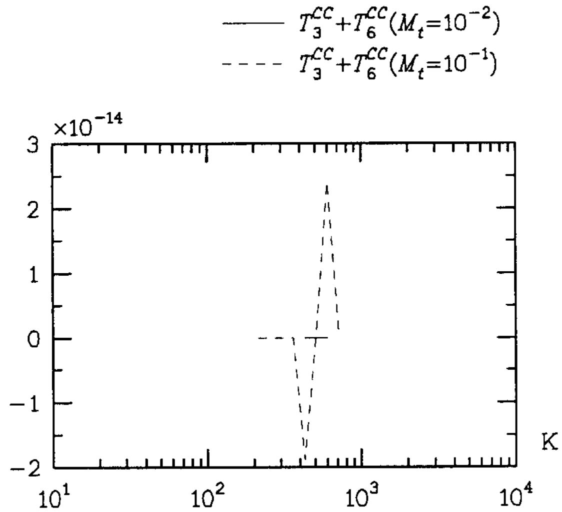


Figure 29:Triadic $T_3^{CC}+T_6^{CC}$ at different low M_t

FIG. 30.

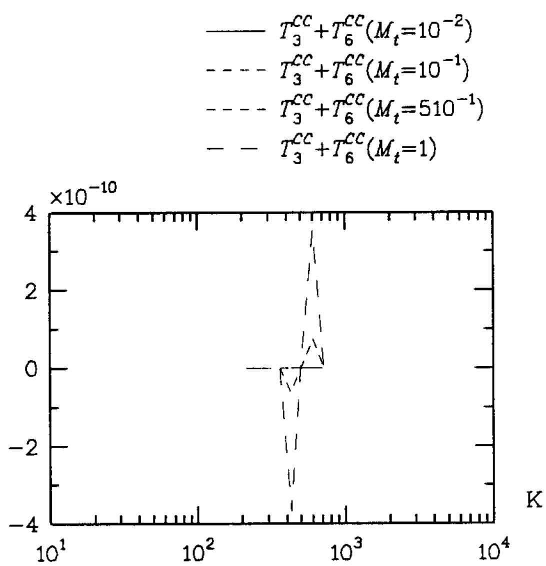


Figure 30:Triadic $T_3^{CC}+T_6^{CC}$ at different M_t

FIG. 31.

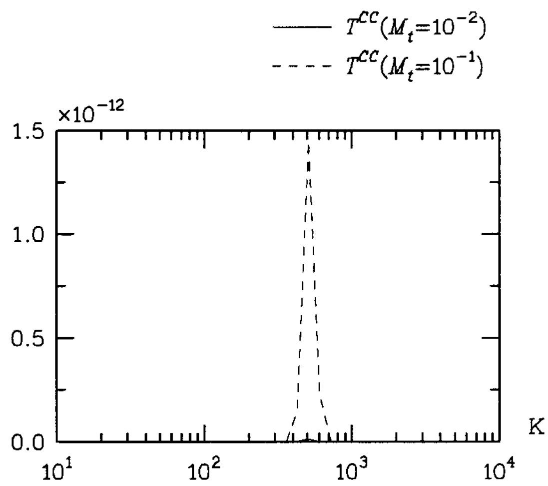


Figure 31:Triadic T^{cc} at different low M_t

FIG. 32.

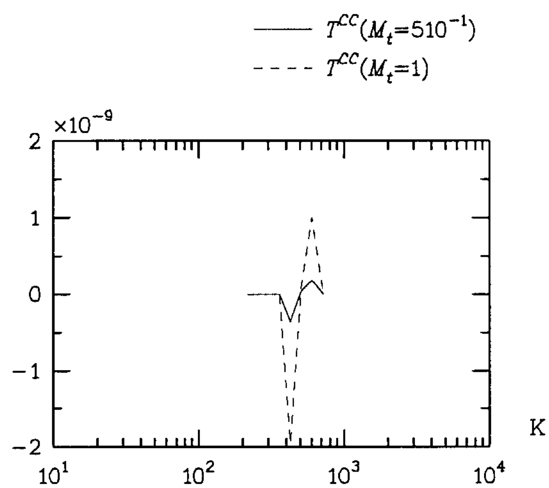


Figure 32:Triadic T^{cc} at different high M_t

FIG. 33.

5. Conclusion. We have investigated the energy transfer process of compressible turbulence using a two-point closure theory, Eddy-Damped-Quasi-Normal-Markovian (EDQNM) closure, as well as the Helmholtz decomposition, a method that separates the compressible and solenoidal modes. We focused on the following issues: (1) What is the mechanism of energy exchange between the solenoidal and compressible modes, and (2) Is there an energy cascade in the compressible energy transfer process? We found that the compressible energy is transferred locally from the solenoidal part to the compressible part. We also found that there is an energy cascade of the compressible mode for high turbulent Mach number ($M_t \geq 0.5$). Since we assume that the compressibility is weak, the magnitude of the compressible (radiative or cascade) transfer is much smaller than that of solenoidal cascade. We confirmed these results by studying the triadic energy transfer function, a most fundamental aspect of the energy transfer.

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